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SHIP MOTIONS IN REGULAR AND IRREGULAR SEAS

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SUMMARY AND INTRODUCTION

The present paper* is written as a record of an informal lecture delivered by the authors at the Office of Naval Research in Washington, D.C., on May 3, 1954. It consists of two distinct parts: the first describes the application of a theory of rigid body motions to a ship moving in a regular head or following sea; the second part deals with the problem of the representation of an irregular storm sea and of the theory of ship motions in irregular seas.

The theory of ship motions in a regular sea, which is presented first, can be considered as the continuation of the work originated by Kriloff in 1896 and represented recently in the most developed form in papers by Weinblum and St. Denis. The new development consists in the introduction of coupling between heave and pitch motions, and in the more complete discussion and evaluation of various coefficients in the coupled differential equations of motions, particularly those of the cross-coupling terms. Recent experimental data obtained at E.T.T. on forcing functions due to waves show them to be much smaller than was previously assumed on the basis of the Froude-Kriloff hypothesis. A comparison of computed and experimentally determined ship motions shows quite good correlation.

In the second part, some significant features of recent theories for the study of ship motions in irregular seas are discussed. It is shown that progress in this phase of seakeeping research does not need to await the complete solution of the problem of motions in simple seas.

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PART I

HEAVING AND PITCHING RESPONSE OF A SHIP TO A REGULAR SEA

Equations of Motion

The obvious difficulty of obtaining quantitative data on the state of the sea and on the resulting ship motions makes it necessary to investigate the subject by means of laboratory research. The problem can be attacked in three different ways:

(a) by direct model experiment in a towing tank;

(b) analytically, using the simple theory of forced oscillations, with independent estimates of the various coefficients;

(c) analytically, treating the subject as a boundary value problem and evaluating the boundary condition at the hull, taking into account both the prescribed wave motion and the (originally unknown) ship motion.

Examples of the last method are found in the recent work of Stoker and Peters (Reference 1), and in the previous work of Haskind translated by the Society of Naval Architects and Marine Engineers (Reference 2). For the present, emphasis will be placed on the second method; the first -- the experimental -- method will be discussed later.

The second method was originated by Kriloff in papers read before the Institution of Naval Architects in 1896 and 1898 (Reference 3), and was followed by Hazen and Nims (Reference 4) and Bull (Reference 5), and by Weinblum and St. Denis in two S.N.A.M.E. papers of 1950 and 1951 (References 6 and 7). The subject and the methods of the present paper can be considered as a direct continuation and development of the last mentioned work.

The case of a simple linear oscillator acted upon by a harmonic forcing function of maximum amplitude F is represented by the following equation:

$$a\ddot{z} + b\dot{z} + cz = F \cos \omega t \quad , \tag{1}$$

the damping force per unit velocity $\dot{z}$, and c is the restoring force per unit of displacement, z. Application of equation (1) to, say, the heaving motion of a ship excited by a simple head sea, considering the upward velocity of the ship and water surface as being positive, results in

$$\left[m\ddot{z} + m_1(\ddot{z} - \ddot{\zeta})\right] + b(\dot{z} - \dot{\zeta}) + cz = (F_d - F_s)\cos\omega t \quad . \tag{2}$$

Here, the inertial term is shown to consist of the mass of a ship, m, multiplied by its acceleration, and the mass of the entrained water, m_1, times the relative acceleration which is the difference between that of the ship, $\ddot{z}$, and of the wave surface, $\ddot{\zeta}$. Likewise, the coefficient of the damping force is multiplied by the relative vertical velocity of ship and wave ($\dot{z} - \dot{\zeta}$). The exciting force is shown to consist of two terms, F_d -- due to changes of displacement -- and F_s -- due to the pressure gradient in the waves, usually referred to as the "Smith Effect."

The present problem is to find the ship motion resulting from a specified wave form, which therefore is assumed to be known. The first step in the solution is to separate the known wave motion and the unknown ship motion terms of equation (2), which leads to

$$(m + m_1)\ddot{z} + b\dot{z} + cz = (F_d - F_s)\cos\omega t + m_1\ddot{\zeta} + b\dot{\zeta} \quad . \tag{3}$$

The left-hand side of the above equation represents now the motion of a ship oscillating in smooth water, and the right-hand side represents the exciting force due to waves acting on a restrained ship which is found to consist of four terms representing forces due to:

change of buoyancy due to wave profile,
change of pressure gradient in the undisturbed wave (Smith Effect),
acceleration of the "added mass" due to wave motion,
water velocity in wave affected by the presence of the ship's hull.

The consideration of only the first two terms on the right-hand side in equations of motion is known as the "Froude-Kriloff hypothesis." In this hypothesis, the pressures acting on the ship surface due to the undisturbed waves are considered, neglecting completely the added water disturbance due to the presence of the ship's hull. The latter two terms in the definition of exciting force represent the effect of this distortion or interference.

Equation (3) is written for the heaving motion and a similar equation is easily written for the pitching motion. In all previous work (except Haskind's), the equations of motion were written and solved separately for heaving and pitching, and the coupling effect between these two motions was neglected. Yet it is known from other fields of activity that the coupling effects can be very important.

A coupled system of pitch and heave motion is represented by a pair of simultaneous differential equations of the usual form:

$$\left.\begin{aligned} a\ddot{z} + b\dot{z} + cz + d\dot{\theta} + e\theta &= \overline{F}e^{i\omega t} \\ A\ddot{\theta} + B\dot{\theta} + C\theta + D\dot{z} + Ez &= \overline{M}e^{i\omega t} \, . \end{aligned}\right\} \quad (4)$$

The first three terms of each equation are identical with the original non-coupled equations. The last two terms on the left-hand side of each equation are coupling terms: $d\dot{\theta}$ is a heaving force due to pitching velocity, $\dot{\theta}$; $e\theta$ is the heaving force due to pitching displacement, θ; and $D\dot{z}$ and Ez are the pitching moments due to heaving velocity and heaving displacement, respectively. The cross-coupling inertial terms are omitted in view of previous experience with airplanes and seaplanes in porpoising, in which cases they were shown to be negligibly small. The harmonic forcing functions on the right-hand sides of equations (4), which are equivalent to the right-hand sides of equation (3), are represented in complex form in order to simplify the subsequent algebraic operations, and it is understood that the real part is to be taken. $\overline{F}$ and $\overline{M}$ are complex amplitudes of forcing functions, i.e., they indicate both the absolute magnitudes of the force and moment, respectively, and their phase relationships with respect to $e^{i\omega t}$. For instance, if a convention is established that time t is zero when the wave crest is at the midship section, and if it is observed that the heaving force has a maximum value of 1 lb. at this instant, then $\overline{F} = F = 1$. However, if the maximum pitching moment of, say, 3.3 ft/lb. occurs 90° earlier in the cycle, i.e., when the wave crest is at Station 5 (considering the ship length to be divided into 20 parts), and the nodal point at the midship section, then $\overline{M} = 3.3i$, where $i = \sqrt{-1}$. In the present work, the forcing functions have been determined as a whole from experiments on a completely restrained model.

Solution of the Equations of Motion

It will be assumed that equations (4) are linear, and that all coefficients are constant and are independent of time. It will be further assumed that the ship is moving in a uniform sea and that a steady state of pitching and heaving is established. In such a case, the transient responses have been damped out, and only the particular solution of equations (4) is needed. Since the forcing functions are harmonic, the resultant motion can be expected to be likewise harmonic, and the solution can be assumed to be in the form

$$z = \overline{Z}e^{i\omega t} \quad \text{and} \quad \theta = \overline{\theta}e^{i\omega t} , \tag{5}$$

where $\overline{Z}$ and $\overline{\theta}$ are complex amplitudes of the form

$$\overline{Z} = Z_o e^{i\alpha} \quad \text{and} \quad \overline{\theta} = \theta_o e^{i\beta} .$$

Here, Z_o is the absolute value of heaving amplitude, θ_o is that of pitching, and $-\alpha$ and $-\beta$ are the phase lags of motion with respect to $e^{i\omega t}$.

Substitution of the assumed solutions (5) into equations (4) leads, after a few simple algebraic operations, to the evaluation of complex amplitudes:

$$\overline{Z} = \frac{\overline{M}Q - \overline{F}S}{QR - PS} \tag{6}$$

$$\overline{\theta} = \frac{\overline{F}R - \overline{M}P}{QR - PS} . \tag{7}$$

The capital letters in the above expressions represent the groupings of the coefficients of differential equations (4) as follows:

$$\left.\begin{aligned} P &= -a\omega^2 + ib\omega + c \\ Q &= id\omega + e \\ R &= iD\omega + E \\ S &= -A\omega^2 + iB\omega + C . \end{aligned}\right\} \tag{8}$$

In this form, the numerical solution of the coupled equations (4) can be carried out quite conveniently. First, the values of all the coefficients of equations (4) are listed, then groupings of those shown by equations (8) are computed, then the complex amplitudes (6) and (7), and from these, finally, the absolute amplitudes Z_o and θ_o and the phase lag angles $-\alpha$ and $-\beta$ of heave and pitch, respectively, are computed.

Evaluation of Coefficients

The solution of the coupled equations of motion (4) is shown above to be quite simple and suitable for the analysis of ship motions. The practical value of the calculations hinges now entirely on whether it is possible to estimate the values of the various coefficients of equations (4). The simplest and most definite coefficients will be treated first: the restoring force and moment coefficients c and C and the cross-coupling coefficients e and E due to changes of displacement in pitch and heave. All of these are in reality quite nonlinear, but with a linearizing assumption they are computed on the basis of the waterplane offsets y as follows:

$$\left.\begin{aligned}
c &= \frac{2\rho g}{z}\int_{-L/2}^{+L/2} yz\;dx \\
C &= \frac{2\rho g}{\theta}\int_{-L/2}^{+L/2} y(\theta x)x\;dx \\
e &= \frac{2\rho g}{\theta}\int_{-L/2}^{+L/2} y(\theta x)\;dx \\
E &= \frac{2\rho g}{z}\int_{-L/2}^{+L/2} yzx\;dx\;.
\end{aligned}\right\}\quad (9)$$

In the above, x is the distance from the midship section, y denotes the half-breadth of the waterplane at a given x, and equations (4) define the motion as that of translation of the center of gravity in heave, and the rotation of the ship about it in pitch. The usual small displacement of the C.G. from the midship section will be neglected here for simplicity, and the origin of coordinates $x = 0$ will be taken at the midship section. The quantity ρg is the weight of water per cubic foot. The arbitrarily assumed displacements z and θ are shown only for completeness of form, but in practical computations they can be taken as unity, so that the integrand has the general form of $yx^n\,dx$, where $n = 0, 1,$ or 2. Following the usual practice in displacement calculations, the integration can be readily performed by Simpson's rule.

In the case of the 5-ft. model of D.T.M.B. Series 60, with a block coefficient of 0.60, the evaluation of the above integrals gives

c = 146 lb. per foot of heaving displacement

C = 202 ft.-lb. per radian of pitching displacement

$e = E$ = -26 lb. per radian or ft.-lb. per foot of displacement.

Next in order of simplicity are the mass coefficients $\underline{a}$ and A. Here, $a = m\,(1 + k_z)$, and $A = J\,(1 + k_\theta)$, where m and J are the mass and the longitudinal moment of inertia of a ship, respectively, and k_z and k_θ are the coefficients of added mass of entrained water or accession to inertia. Following the common practice, these are assumed on the basis of comparison with submerged ellipsoids, and in the present work were actually read from the charts given in the Weinblum and St. Denis S.N.A.M.E. paper of 1950 (Reference 6). At this point, it is well to remember that the natural angular frequencies of oscillation of a ship in still water are given by the expressions

$$\nu_z = \sqrt{\frac{c}{m(1 + k_z)}} \quad \text{and} \quad \nu_\theta = \sqrt{\frac{C}{J(1 + k_\theta)}} \quad . \tag{10}$$

Since the restoring force and moment coefficients are known from equations (9), the correctness of the assumed coefficients of accession to inertia can be verified by comparison of the computed and experimentally observed natural periods. The correlation in this case was found to be very good, and appears to be independent of model forward velocity.

Next are the four coefficients b, B, d, and D of the terms which are dependent on the velocity of motion. The basic building block in evaluating all of these is the expression derived by St. Denis in his 1951 S.N.A.M.E. paper (Reference 7):

$$N(x) = \frac{\frac{1}{2} \rho \omega \sin^2(ky)}{k^2} e^{-2ka} , \tag{11}$$

where

N(x) is the force acting on a unit length of a prismatic floating body due to a unit heaving velocity, $\dot{z}$; this expression was derived by considering the dissipation of energy in plane waves caused by the heaving oscillations of a prismatic body of the beam 2y and the mean draft <u>a</u> (here, the frequency of oscillation is equal to the frequency of encounter with the sea waves);

y is the offset at the load waterline (a function of x);

k is wave number $2\pi/\lambda$;

λ is the length of the waves generated by the ship's oscillation of circular frequency, ω ;

a is the mean depth of the section of the ship considered, i.e., (sectional area)/2y.

Various coefficients for the entire ship are now evaluated by the strip theory method in which it is assumed that each element of ship length acts independently of the adjacent ones, as though it were a part of an infinitely long prismatic body. The contribution of all elements are then integrated by Simpson's rule. The simple integrals are:

$$b = \int_{-L/2}^{+L/2} N(x)\, dx \tag{12}$$

in lb. per ft./sec. of heaving velocity, $\dot{z}$

$$B = \int_{-L/2}^{+L/2} N(x) x^2\, dx \tag{13}$$

in ft.-lb. per rad./sec. of pitching velocity, $\dot{\theta}$

$$d = D = \int_{-L/2}^{+L/2} N(x)x\,dx \qquad (14)$$

in lb. per rad./sec. of pitching velocity, $\dot{\theta}$, and in ft.-lb. per ft/sec. of heaving velocity, $\dot{z}$.

The curves representing the computed values of the above integrals plotted vs. circular frequency of encounter, ω, are given on Figure 1.

In the case of the coefficient b, the largest part of its value is contributed by the middle part of the ship's length in which the water flow is not too different from that of a long prismatic body. Whatever small deficiency of force results from the cross flow from one ship section to a very similar neighboring one is compensated for by the addition of a viscous or eddymaking force resulting from the water flow around the bilges. The coefficient b can therefore be assumed to be correctly represented by the integral (12).

In the case of the integral (13), for the moment coefficient B, a different situation exists. The largest part of the contribution to the value of this integral comes from sections at the ends of the ship where the true three-dimensional flow differs considerably from the two-dimensional flow postulated by equation (11) and by the integral (13); therefore, the true value of the coefficient B must be much smaller than that indicated by the integral. The exact three-dimensional solution is available in the case of an elongated submerged spheroid (References 8 and 9), and comparison of it with the strip theory integral (13) indicates that one-half of the value of the latter must be used. It is assumed in the present work that the same factor of one-half will also be applicable in the case of surface ships.

Integral (14) for the cross-coupling coefficients d and D contains x in the first power in the integrand as against zero power for b, and square for B. It will be assumed, therefore, that 75% of the calculated value of the integral (14) is to be used as compared to 100% and 50% for b and B, respectively. It will later be shown that, with these assumptions, there is very good correlation between the computed and experimentally measured amplitudes of ship motions.

The Forcing Functions

It remains now to consider the forcing functions on the right-hand side of equations (4), which, in the case of heave, for instance, are equivalent to the right-hand side of equation (3). The first two terms of equation (3), due to displacement changes and changes of pressure gradient, comprising the "Froude-Kriloff hypothesis," are readily calculable by the familiar methods of buoyancy calculation. It is believed that the other two terms can also be computed by methods indicated in the St. Denis S.N.A.M.E. paper of 1951 (Reference 7), but this work has not yet been done. Instead, the entire forcing function was measured experimentally by supporting a restrained model in the towing tank by means of dynamometers having small deflections, since the first part of the problem is defined as that of verifying the correctness of the use of the coupled equations (4), and of the evaluation of their coefficients. Suffice it to say here that the total heaving force was found to be of the order of 50%, and the total pitching moment of the order of 75% of those computed on the basis of the Froude-Kriloff hypothesis. The measurement of these forcing functions was made under the sponsorship of the H-7 panel of the S.N.A.M.E; the preliminary data are given in Reference 10.

Comparison of Calculations with Experimental Results

So far, the calculations have been applied to one model of D.T.M.B. Series 60, having a 0.60 block (Reference 11), at one wave length equal o the model length and a height of 1/48 of its length. Since this is the wave height chosen by the International Conference on Ship Hydrodynamics for comparative tests of models in many tanks, a large amount of experimental data should soon become available for comparison with theoretical calculations. Four American laboratories have kindly furnished their test data to the authors. Figure 2 shows the computed curve of pitch amplitudes, and the experimental data of the two laboratories which were most consistent both with each other and with the computed curve. It will be observed that good agreement is obtained throughout the speed range from zero to a speed-length ratio of nearly 1.0. This good correlation throughout the speed range can be taken as a confirmation of the validity of the computation of the coefficients of the velocity terms shown in Figure 1, as well as the application of correction factors of 1.0, 0.75, and 0.50 for coefficients b, c and D, and B, respectively.

A peculiar feature of the experimental data, the occurrence of the maximum amplitude of pitch at very low speed, is also confirmed by the calculated curve. The natural period of the model oscillation in pitch indicates the resonant speed to be at about 1.8 ft./sec., whereas the apparent resonance of the actual motion is shown to be at about 1.5 ft./sec. This shift of the apparent resonance to a lower speed is now seen to be the effect of the cross-coupling coefficients d, D, e, and E. A different amount, and possibly a different direction of shift, can be expected with different values of these coefficients for another hull. The usual characteristics of conventional ship forms makes one expect, however, that a shift of resonance to a lower speed will be generally found as in the present case.

Figure 3 shows a similar correlation for heaving amplitudes. It can be seen that the disagreement of the experimentally measured heave, even in the case of the two laboratories having the closest agreement of data, is very large, and therefore a complete verification of the computational procedure is not possible. Nevertheless, good agreement as to the order of magnitude of the maximum heaving amplitude is evident, as well as general similarity of the shapes of the computed and experimental curves. The experimental data indicate, however, the shift of the apparent resonance in heave to a much higher speed, i.e., in the direction opposite that of the pitching resonance. The computed curves show correctly the direction of the trend in separating the speeds of apparent resonances in pitching and in heaving, but fail to show the shift of heaving resonance to such a high speed as is shown experimentally.

Figure 4 shows the phase relationships. The upper part of the Figure indicates the phase lags of the heave and pitch amplitudes with respect to the amplitudes of heaving force and moment. The maximum heaving force occurs when the wave crest is close to the midship section. The maximum amplitude of heave, however, is delayed until the wave crest rolls further aft. For instance, at the time of apparent resonance in heave, at a speed of 1.6 ft./sec., the phase lag is 90°; in the present case, this is equivalent to 1/4 of the ship's length. The maximum height of the heaving motion will therefore occur when the wave crest is at Station 15, and the following wave trough is at Station 5.

The lower part of Figure 4 shows the lag of heaving amplitudes with respect to pitching amplitude. The experimental data on phase relationships are not yet available, but some preliminary information indicates that the low value of this lag at low speed is found experimentally also. It can be added here that, in the past, few experimental data on ship motions have been published, and then often only in the form of heaving and pitching data separately, without any information on the phase relationship between the two. In such form, the information given does not describe completely the ship motions, and does not permit the derivation of other properties of motion, as, for instance, the vertical acceleration at the bow or stern. In order to completely describe the motion, all three quantities -- the amplitudes of heave and of pitch, and the phase relationship between the two, are necessary. These three quantities will describe the absolute motion of a ship, but they still do not permit judgment of the degree of bow or stern immersion. For this purpose, it is necessary to relate the ship motion to the wave motion by giving these phase relationships also, as shown by the computed ones in the upper part of Figure 4.

A question can be asked at this stage as to what has been gained by the use of the coupled equations of motion as compared to the simpler separate equations for pitch and heave. It has already been mentioned that coupling has brought the calculated resonance in pitch to a lower speed, correctly representing the experimental data. Comparative coupled and uncoupled calculations were made at one speed of 2.1 ft./sec., which is somewhat above the uncoupled resonance speed, with the following results:

	Computed from		Experimental Values
	Uncoupled Equations	Coupled Equations	
Heave Amplitude	0.42 in.	1.1 in.	about 1.15 in.
Pitch Amplitude	10.9°	6.2°	about 6.3°

The high damping in heave and small magnitude of the measured heaving force indicate a very small heaving amplitude, if computed as uncoupled motion, leaving the large experimental heave utterly unexplainable. On the other hand, low damping and high exciting pitching moment tend to give an exaggerated pitching motion when calculated in the single mode. The effect of taking the coupling into account is to hold down the pitching and to increase the heaving, thus bringing both to a much better correlation with the experimental data.

It is interesting to note here that in all cases of previously published calculations of ship motions, such as those of Kriloff in 1898 (Reference 3), Hazen and Nim (Reference 4) and Bull (Reference 5), no experimental verifications of the calculations are made. The indicated pitching motions, however, appear by examination to be exaggerated. This exaggeration is now seen to be the result of two factors: the use of the Froude-Kriloff hypothesis leads to exaggerated values of forcing functions, and the use of uncoupled equations likewise tends to exaggerate the pitching amplitude. In the case of the St. Denis paper of 1951 (Reference 7), the calculated amplitudes do not appear to be abnormal. In the Introduction to that paper, it is stated that the Froude-Kriloff hypothesis was used, but in reality, in the development of the paper, all terms of equations (2) and (3) of the present work were taken into account. Hence, the exaggeration of forcing functions was probably avoided. The exaggeration of the pitch due to the use of the uncoupled equation of motion was likewise balanced by not considering the reduction of the damping coefficient B due to three-dimensional flow. In fact, the assumed damping was effectively taken as double the probable actual one. However, since no experimental check was available in this case either, a more detailed discussion is not possible.

It must be clear from what has been said above that a verification of a theory at one test speed for either pitch or heave alone is quite meaningless. If a theory is to be valid, it must show a reasonable agreement in pitch and heave simultaneously throughout the entire practical speed range. If this stringent requirement is fulfilled, as it appears to be in the present case, the agreement can hardly be called fortuitous, and can be accepted as a valid verification. Further verification is needed in the present case for waves of other lengths and for ships of other forms, but at the moment it appears that a practical engineering tool is near at hand for the prediction of ship motions on the basis of ship hull geometry, and a known or specified sea of simple form. The extension to irregular seas will be discussed in the second part of this paper.

Concluding Remarks

In concluding the first section of this paper, the bearing of the theory on the interpretation of experiments can be briefly discussed. Theoretical calculations indicate that the effect of the heave and pitch coupling is to hold down the pitching amplitudes, or it may be thought of as effectively increasing the damping in pitch. This makes the pitching motions less sensitive to odd disturbances, and, indeed, the experimental pitching data are found to be reasonably consistent. On the other hand, the effect of coupling is to increase heaving amplitudes, i.e., effectively, to decrease the damping in heave, making it much more sensitive to unforeseen excitations. In fact, the experimental discrepancy in the heave data, even between the laboratories having the best agreement, is disappointingly high.

So far, only the coupling between pitch and heave has been considered theoretically. It is extremely desirable to consider the entire motion in a plane of symmetry, i.e., surge as well as heave and pitch, and particularly to investigate the effects of surge coupling and of variation in test conditions in regard to surging motion. This will indicate to what extent it is necessary to provide the correct freedom in surge in laboratory tests. At present, although efforts have been made to have the test conditions correct with respect to heave and pitch, apparently no control is exercised as to the masses involved in surging motion. It would be of interest to check, both experimentally and by calculation, the extent to which this may affect the heaving motion, in which the discrepancies are most conspicuous.

PART II

THEORETICAL AND EXPERIMENTAL INVESTIGATION OF SHIP MOTIONS IN IRREGULAR, LONG-CRESTED SEAS

Importance of Irregular Waves

It is generally recognized that regular waves of the type discussed in Part I of this paper are not representative of actual storm seas, although they may approximate a smooth regular swell. Of course, swell conditions are often encountered at sea after a storm has passed or as the result of a distant storm; they are more common on some of the ocean routes than on others. But the problem which is usually more serious, and certainly more difficult, is that of ship motions in storm seas while a strong wave-generating wind is blowing. The sea is then characterized by great apparent irregularity, incessant change of appearance, and by the fact that waves are "short-crested", i.e., looking along a crest, one may see that it seems to disappear at a short distance, perhaps in a hollow of another wave. The motion of a ship in such a sea likewise is quite irregular, with pitching, rolling, yawing, etc. all combined. Concentrating on pitching, for example, the continually changing apparent periods, lengths, and heights of the waves result in continually fluctuating periods and amplitudes of pitching. Thus, it is clear that the use of average waves for the study of motions is inadequate, and the present knowledge of behavior in regular waves is not directly applicable to the problem of motions in irregular storm seas.

A very promising approach to the irregular sea motion problem has been set forth by St. Denis and Pierson in Reference 12, and in the work of Fuchs and MacCamy (Reference 13). The essential idea is that motions in irregular waves can be accounted for on the basis of motions in regular waves of a variety of lengths. It is first shown that the irregular surface of the ocean in a storm can be considered as the combined effect of an infinite number of component waves of different length, height, and direction. The hypothesis is then advanced that ship responses to such irregular waves in pitch, heave, roll, etc. may be considered as the sum of the responses to each of the component waves. This means that studies of regular waves become more, rather than less, important than in the past.

The theory has far-reaching consequences, for, if confirmed, it means for one thing that when satisfactory analytical solutions for ship motions in regular waves are obtained, they may be applied at once to obtaining the complete analytical solution of the irregular sea case. However, this goal is not within immediate possibility of achievement. A second consequence is of more immediate importance: the possibility of obtaining the ship responses to regular waves experimentally, and applying them to the irregular wave problem without waiting for the complete analytical solution. It is this second aspect that will be considered here.

To start with, it must be realized that successful completion of any research requires a certain simplification of the problem. On the one hand, it must be stripped of unimportant elements, so that attention can be concentrated on the most significant features, and on the other hand, certain important features must be segregated for investigation first, independently of the others. For the present problem, the complete ship motion is separated into the three component rotations of pitch, roll, and yaw, and the three translations of heave, sway, and surge. Experience with ship motion studies -- as well as with comparable work in aeronautical engineering -- has shown that it is feasible to segregate the three motions in the plane of symmetry -- heave, pitch, and surge -- and to investigate these first, postponing the study of all six until later. Physically, this means that motions in head or following seas can be investigated independently of motions in waves of other directions. This is clear enough in a regular wave or swell, but in the case of an irregular sea, it implies the further assumption that all irregularity in the transverse direction is neglected. In short, the problem is considered as being completely two-dimensional. This type of two-dimensional, or long-crested, irregular sea will be further discussed later. Suffice it to say at this point that the current development of the analytical and experimental techniques for ship motion studies is not yet adequate to deal with the problem satisfactorily unless so simplified or limited.

Characterizing the Ocean Surface

The question of ship motions in irregular seas leads first to the problem of characterizing the complex surface pattern of the ocean. It is comparatively easy to create a completely confused sea in the model tank, and this has been done by the use of baffles, allowing reflections from the ends and sides of the tank, and by introducing irregular paddle motions. None of these methods is desirable, for the resulting confusion is not like the irregularity of the sea, and it is not subject to satisfactory analysis or repetition.

The significance of recent work in physical oceanography (Reference 14) is that the confusion of a storm sea is found to be more apparent than real and that the application of statistical methods can bring a certain degree of order out of the apparent chaos. These methods received their initial impetus from the need during the last war for predicting surf conditions at landing beaches. Work has been continued in this country under the sponsorship of the Beach Erosion Board and the Office of Naval Research, leading to the extension of the knowledge and techniques for application to ship motion problems (see Reference 12).

Sample records of the ocean surface at a fixed point are shown in Figure 5. Such records are obtainable from a pressure recorder on the bottom in shallow water, for example, or by means of a long floating "wave pole" in deep water, with instruments for recording the water elevation in relation to it. The first record (a) is typical of the sea surface in a storm area, where the wind has been blowing for some length of time. It is characterized by an appearance of great irregularity and confusion, with wide fluctuations in the intervals between crests and in the wave heights. Record (b) is typical of a "swell," the sea surface after a storm has passed or at some distance from a storm. It is irregular, but not nearly so irregular as the other record.

Both records of Figure 5 -- even the apparently chaotic first one -- are found to be amenable to analysis statistically because of a very important observed fact. To explain this fact, the record is first marked off at equal small intervals of time, as shown in Figure 5. and the deviations of the points from the average line are classified and plotted on the basis of frequency of occurrence. It is found then that the result is very close to a typical "normal" or Gaussian distribution curve, as shown in Figure 6. The distribution curves will be different for different wave records, but they are always found to be very close to the typical shape. Starting from this important characteristic of ocean wave records, Pierson (Reference 14) has found that the sea surface can be represented as an infinite number of infinitesimal sine waves superimposed in random fashion, so that all of the crests never coincide. As a practical matter, the elevation at any instant may be considered as the sum of points on a large number -- instead of an infinite number -- of sine waves of very small amplitude. This picture may be visualized by assuming a large number of corrugated plates, each with a different sized corrugation, stacked on top of one another. The composite wave pattern can then be obtained by adding the heights of the points on the plates vertically in line. Each of the component waves possesses the well-known characteristics of simple surface waves whereby the wave is completely specified by its frequency (or period) and its height, since wave length and velocity are known functions of frequency (or period). However, these component waves are not directly visible in a seaway or a record; they can be found only by a rather complicated analytical method.

The frequencies or periods of the component waves and their relative importance in an actual wave record are described by the "energy spectrum" of the seaway. For the case of a reasonably steady wind blowing over an initially calm sea for a sufficient length of time in the open ocean, Neumann (Reference 15) has obtained a mathematical expression for this spectrum, which can be worked out for any desired wind velocity. A typical example is shown in Figure 7(a), in which the curve simply indicates the relative amplitudes or heights of the many wave components present in a typical ideal seaway. In this Figure, it is shown how the wave pattern is approximated by taking the sum of say 14 component waves, neglecting the low waves at each end. The frequency, and hence length, of each component is indicated by the position of one of the narrow rectangles along the ordinate scale of frequencies, ω (or period, T), and the amplitude of each component is given by the square root of the area of the same rectangle. Figure 7(b) shows the 14 components corresponding to each of the rectangles -- the "corrugated plates" previously mentioned. Summing up the heights of these components at successive instants will give a typical wave pattern similar to Figure 5(a). The greater the number of components taken, the more exactly will the pattern approximate a possible ocean wave record. However, no two records -- either artificial or real -- will ever be exactly alike, even though their statistical properties may be identical.

Thus, the apparent irregularity of the sea and of ship motions in it may be considered simply to be the sum of a very large number of regular waves and motions superimposed.

The energy spectrum is different from the ideal, however, if the wind has not blown long enough for the sea to be fully developed, or if the open sea distance or "fetch" over which the wind has blown is limited. The spectrum grows from the high frequency end, and, in incomplete form, it would terminate at some frequency -- say ω_7 in Figure 7 -- determined by the duration or fetch. Hence, the spectrum of a simple storm appears to be a fairly definite function of wind velocity, duration, and fetch.

So far, no mention has been made of the direction of motion of the waves. Actually, it appears that the wave components are not all travelling in the same direction -- as a result of fluctuations in the storm winds which created them; that is, the corrugated plates previously mentioned lie at different angles to one another. This results in the characteristic "short crestedness" of ocean storm waves in a direction at right angles to their motion. However, it is believed that the directions of most of the components lie within $\pm 30^{\circ}$ of that of the dominant crests -- when the disturbance created by a single storm is considered. The theoretical form of three-dimensional spectra has been tentatively worked out, thus permitting the mathematical representation of the sea surface over an area as well as at a fixed point (see Reference 12).

The work discussed has dealt entirely with an idealized spectrum of a single, simple storm. An actual sea spectrum may be much less smooth and regular than the spectrum shown in Figure 7. Many more observations at sea are needed, using the statistical theory as the basis for analyzing the resulting data. Meanwhile, the idealized spectrum appears to be quite satisfactory for the present analytical and experimental work. Even though actual storms may differ somewhat, they will certainly be of the same general character.

Significance of Short-Crestedness

If the study is limited first to the symmetrical motions of pitch, heave, and surge, as discussed above, the next step is to determine how important the characteristic of short-crestedness is to the present problem. Short-crested beam seas, containing components of many different directions, obviously produce greater longitudinal forces and moments than long-crested seas, no matter how irregular, if parallel to the ship's course. Hence, for this case, short-crestedness is very important. As the ship's heading is shifted more and more toward right angles to the dominant direction of the sea, the effects of the variation in direction will become less and less. In order to determine exactly what the effect would be in a typical case of a head sea, some calculations were carried out on the basis of the three-dimensional spectrum given in Reference 12. The results are shown in Figure 8.

One of the curves in Figure 8 shows Neumann's ideal storm spectrum at a fixed point, corresponding to a 30-knot wind (Reference 16). When taking a record at a fixed point, it makes no difference whether or not a long- or short-crested sea produced the record. However, a ship moves through the sea, and therefore it is convenient to consider the spectrum which would be recorded at a moving point. In this case, it does make a difference whether or not the sea is long- or short-crested. The spectrum for the long-crested case is obtained by shifting points on the original spectrum so as to correct for the period of encounter. The spectrum corresponding to a point moving at 15 knots in a direction opposite that of the waves is shown in Figure 8. To obtain it, values of the relative amplitude $[r(\omega)]^2$ for different values of ω have simply been replotted at the ω value corresponding to the frequency of encounter.

Finally, the corresponding spectrum was determined for a point moving at 15 knots into the wind, with a three-dimensional spectrum of the form given in Reference 12 for an ideal short-crested sea. This was done by assuming the short-crested sea to be made up of groups of components differing from each other by headings of 10°. The spectrum for all components at each different angle was then calculated, corrected for the effect of speed on frequency of encounter, and integrated by equation 1.23 of Reference 12. The result is also plotted in Figure 8.

It may be seen that the difference between the two spectra for a moving point is very small, and hence the use of a long-crested sea for experimental and analytical work on symmetrical motions is certainly justified. As a matter of fact, the two curves could be made practically to coincide by using a fictitious wind velocity for the long-crested sea somewhat higher than the specified 30 knots. This would give a sea almost exactly equivalent to the short-crested sea, insofar as the spectrum is concerned. This refinement, however, is hardly necessary in view of the degree of accuracy with which sea spectra are known.

It is important also to consider wave lengths when comparing different irregular seas in relation to ship motions. The spectrum characterizes the periods and heights of component waves, but great care is required to obtain information regarding the lengths of the component waves. The difficulty is that periods or frequencies are changed by movement of the point of observation, but effective wave lengths are changed only by differences in direction. When a ship encounters a short-crested head sea, the effective length (in relation to symmetrical ship motions) of each component wave is the actual length divided by the cosine of the angle between its direction and that of the ship. Hence, it is clear that the average length of a short-crested head sea must be somewhat longer than that of a long-crested sea having the same spectrum at a fixed point. It would be possible to work out the average wave length for the long- and short-crested seas, but it will suffice to note in this case that if a somewhat higher fictitious wind velocity were assumed in order to bring about agreement between the two spectra in Figure 6, then, at the same time, the average wave lengths would be brought into closer agreement. In the case of following seas, on the contrary, the corrections to spectrum and to wave length work in opposite directions.

The significance of all this is that if a long-crested sea is adopted for studies of symmetrical motions, a legitimate simplification is obtained and not merely a rough approximation having no theoretical foundation. It simplifies both analytical and experimental problems greatly, and opens the way to the possibility of a straightforward verification and evaluation of the St. Denis-Pierson theory (Reference 12).

In simplifying the problem of ship motions by neglecting short-crestedness as described above, it must not be forgotten that there are conditions of the sea in which [illegible] directions are very important to the symmetrical motions. These are cases which cannot be approximated by a single simple storm, such as

(1) the combined effect of two or more storms at some distance apart;

(2) the effect of a storm sea building up on an existing swell (although, in time, the swell may be destroyed by the storm wind (Reference 16), the combined effect may be apparent for some time);

(3) the effect of circular storms or sudden shifts of wind direction (Reference 17); conditions at the eye of a hurricane, for example, defy analysis by present methods.

Conditions such as the above can produce very extreme cases of short-crestedness, with pyramidal crests and deep holes. Data on such conditions are certainly needed, and the corresponding ship motions must in due course be studied analytically and experimentally.

Creating Irregular Long-Crested Waves in a Model Tank

There are two possible methods of creating irregular long-crested waves in a model tank. One is to superimpose a finite number of components, a process which becomes increasingly complex as the number of components is increased. The other is to vary continuously the frequency and amplitude of the wave-generating mechanism. A variation of the second method looks particularly promising at the S.I.T. at the present time. A sample record of the irregular wave produced is shown in Figure 9.

The analysis of typical experimental records has shown that the distribution of points at successive time intervals are very nearly Gaussian, as required, and that the distribution of apparent periods can, by suitable control, be made to correspond closely to the distribution observed at sea, as given by Neumann (Reference 16). A comparison of the period distributions for a typical case is given in Figure 10. "Apparent period" is defined here as the time between successive crests of the irregular wave record, which of course gives no direct indication of the periods of the component waves. An energy spectrum analysis of a $1\frac{1}{2}$-minute record is now being made on the Univac machine at the David Taylor Model Basin, but it is not yet available.

The irregular wave method being developed at the E.T.T. is very simple in principle. Use is made of floats at the wavemaker end of the tank to control wave amplitude, and of a continually varying paddle frequency to provide a wide range of wave frequencies. The object is to make the period of each paddle stroke different, and hence the motor speed control rheostat is changed manually after each stroke. Of course, without some control of amplitude, the short strokes produce higher waves than the long ones, and an unrealistic sea results. By using a light "polarfoam" float, 36 in. long and 4 in. thick, extending the width of the tank, the heights of all waves less than 36 in. long are reduced almost to the height of the 36-in. waves. The exact proportions, number, and ballasting of the floats to give the best results requires further study, but the basic idea seems to be sound.

It would undoubtedly be possible to apply the above method to the operation of a random-sea generator, thereby producing a different wave pattern every time it is operated. This is not the aim, however, because an important feature of the method is that it appears to lend itself to making a reproducible sea. The objective is to provide mechanical-electrical control whereby the wavemaker will repeat at will a sequence of strokes of approximately 3 minutes' duration. This corresponds to about 30 minutes of a full-scale ocean record, which, as indicated by Pierson's work, should provide a satisfactory sample from the statistical point of view (Reference 14). If this reproducibility is successful, it will permit one or more wave patterns to be analyzed once and for all, and thus eliminate the need for multiple analysis of wave records. It is also hoped that direct comparative tests of different models can be made in the same wave pattern for either head or following seas. Of course, a model run in the tank usually requires much less than 3 minutes, and therefore a large number of runs will be required in different parts of the wave pattern.

It must be clear from the first part of this paper, however, that ship motions are governed by a large number of characteristics (expressed as coefficients of the coupled differential equations), and that it is not probable that the best combination of these for improved ship performance in waves can be obtained by pure cut-and-try methods. Further development of analytical methods will also have to be pursued for irregular as well as regular seas. In the irregular case, References 12 and 13 represent the pioneering work; but the first is completely devoid of experimental verification, while the experimental part in the second one is rather brief. It will be necessary, therefore, to devote a considerable amount of effort to the experimental verification and further analytical development of the theory of calculating the complex sea motions from the known ship responses to simple seas.

The experimental application of irregular waves to the problem of bending moments in a ship model will be described in a forthcoming paper in preparation by one of the authors (Reference 18).

The Present O.N.R. Project at the E.T.T.

The work on irregular waves now being carried out at the Experimental Towing Tank is being done under contract with the Office of Naval Research, under the technical cognizance of the David Taylor Model Basin. Briefly, the project has two aims:

(1) to make a direct comparison of the resistance and motions of two extreme hull forms in both regular and irregular long-crested tank waves; and

(2) to check on analytical methods of calculating ship responses to irregular waves on the basis of the responses to regular waves of a wide range of frequencies,

  (a) by means of statistical methods (St. Denis-Pierson, Reference 12), and

  (b) by point-by-point calculation (Fuchs-MacCamy, Reference 13).

It is expected that the statistical study, item 2(a), will yield the motion spectra and the average and maximum amplitudes of motion. The point-by-point calculation, item 2(b), will result in the time history of motion in relation to the wave, in order to provide a picture of wetness of decks, propeller emersion, slamming, etc. It will be noted that the problem of ship motions in regular waves, instead of becoming secondary to the problem of irregular wave motions, now becomes an essential part of it. The simple sea studies provide the necessary building blocks from which it is hoped the irregular sea motion can be constructed.

Work to date on this project has been on problems of instrumentation and background theory, and therefore there are no concrete results to report. However, the indications are that the new theory accounts for sea observations in a qualitative way. Whichever type of motion is considered -- for example, rolling or pitching -- the ship seems to select the resonant frequencies from the broad band of frequencies present in the seaway and to respond most violently to them. This is illustrated in Figure 11, which is based on two Figures in Reference 13. It can be seen that the calculated response spectrum involves a radical shift of the sea spectrum, with a peak at the natural pitching frequency. This is confirmed by sea observations (Reference 19). Insofar as pitching is concerned, normal ship proportions seem to be such that, at ordinary speeds in head seas, ships experience synchronism with the longer wave components, which results in erratic but occasionally violent pitching. At reduced speeds or hove to, conditions are usually ameliorated by the fact that synchronism occurs only with the shorter waves which produce comparatively small pitching moments.

Conclusions

The above considerations suggest that improvements in the seakeeping qualities of ships may be sought in the following directions:

(1) Radical shifts of natural frequencies of oscillation, in order to avoid synchronism as much as possible, through modifications to proportions or weight distribution. In the case of pitching and heaving, shorter periods are distinctly beneficial in this respect.

(2) Changes in hull form or the use of auxiliary devices which will reduce motions when resonance with certain wave components is unavoidable.

(3) Changes in hull form or the use of auxiliary devices which will improve the phase relationship between the waves and the ship motions.

It is hoped that the work on motions in irregular seas being undertaken at the E.T T. will not only clarify the theory involved but will shed some light directly on the above possible improvements in the seakeeping qualities of ships.

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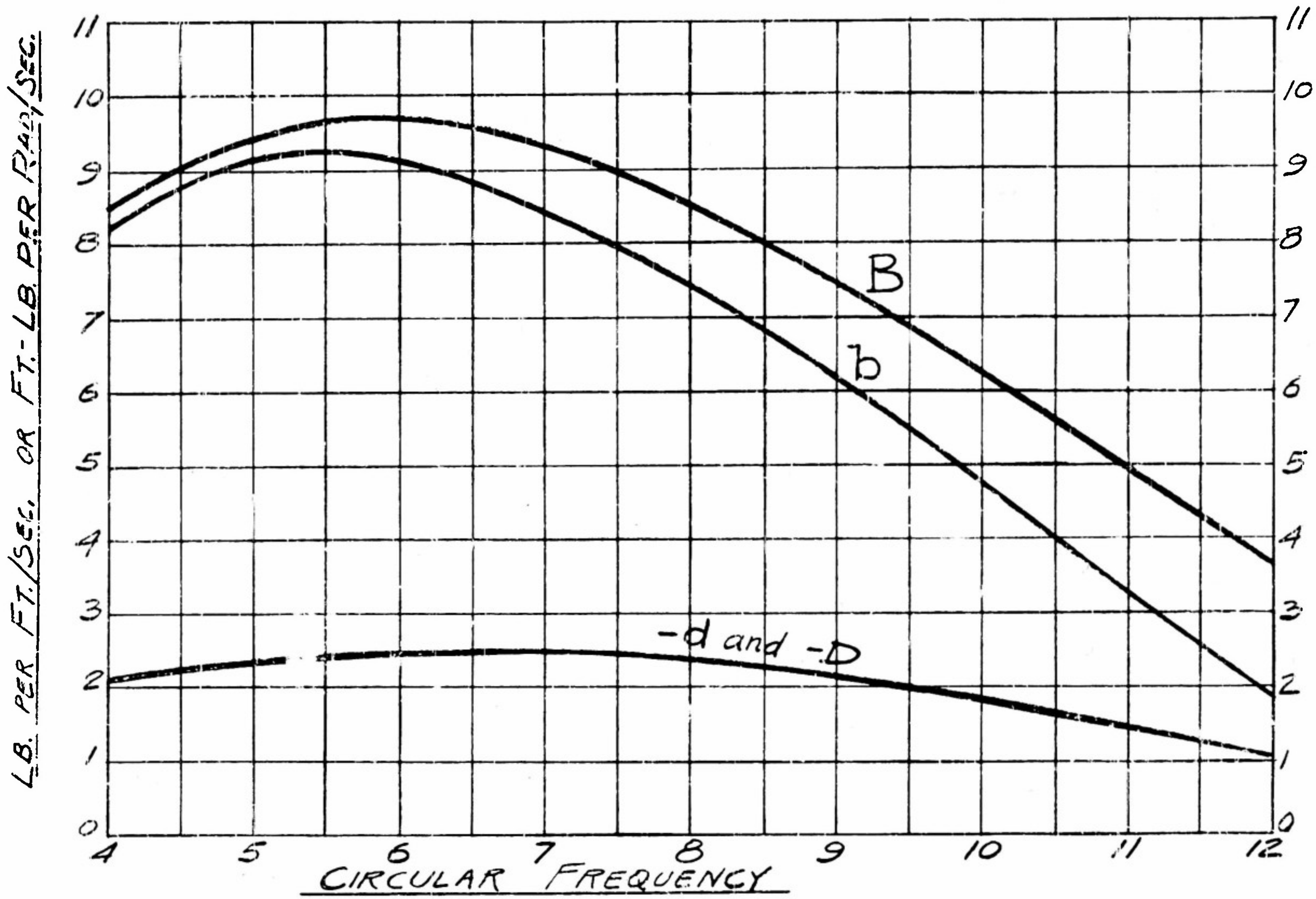
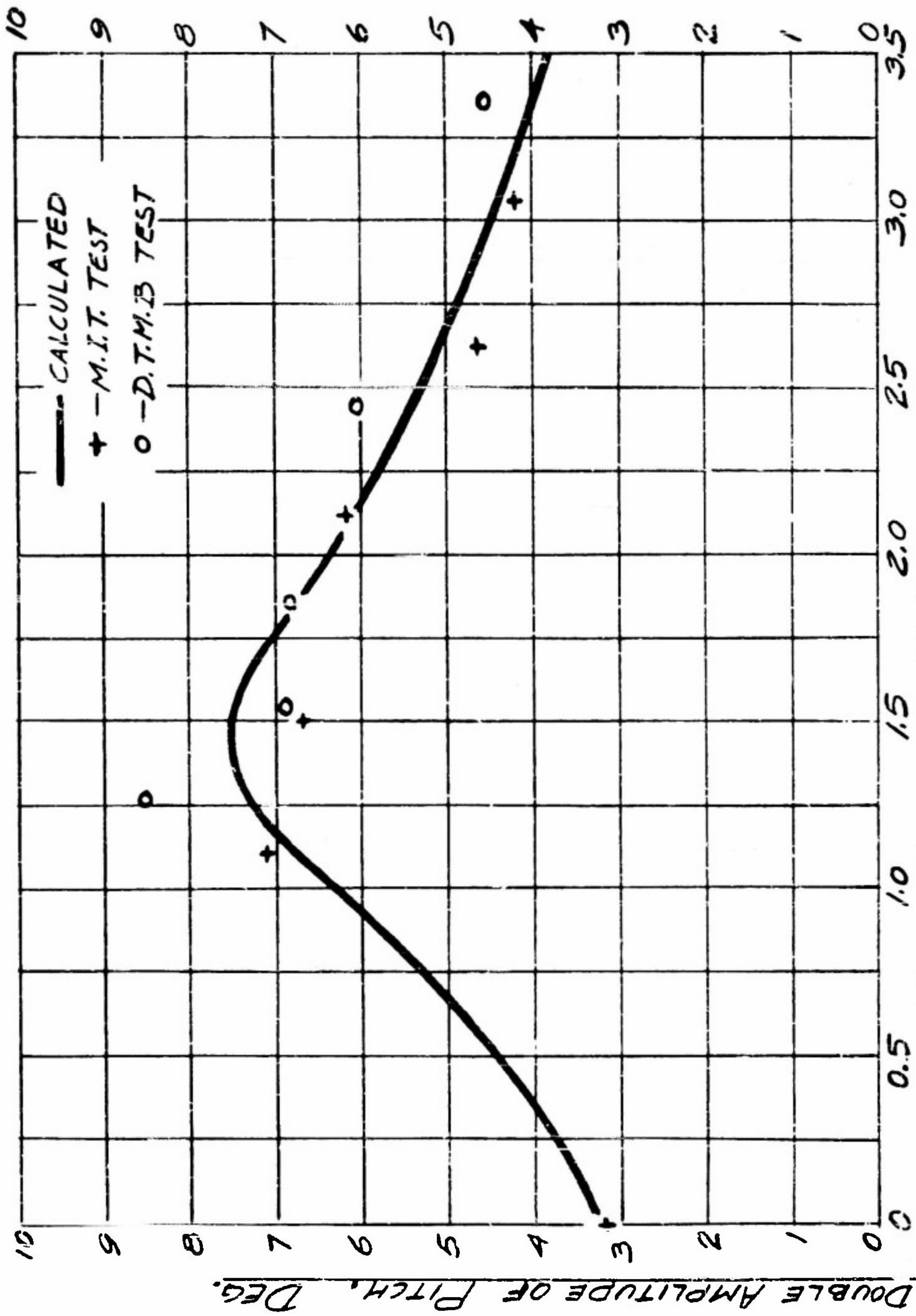
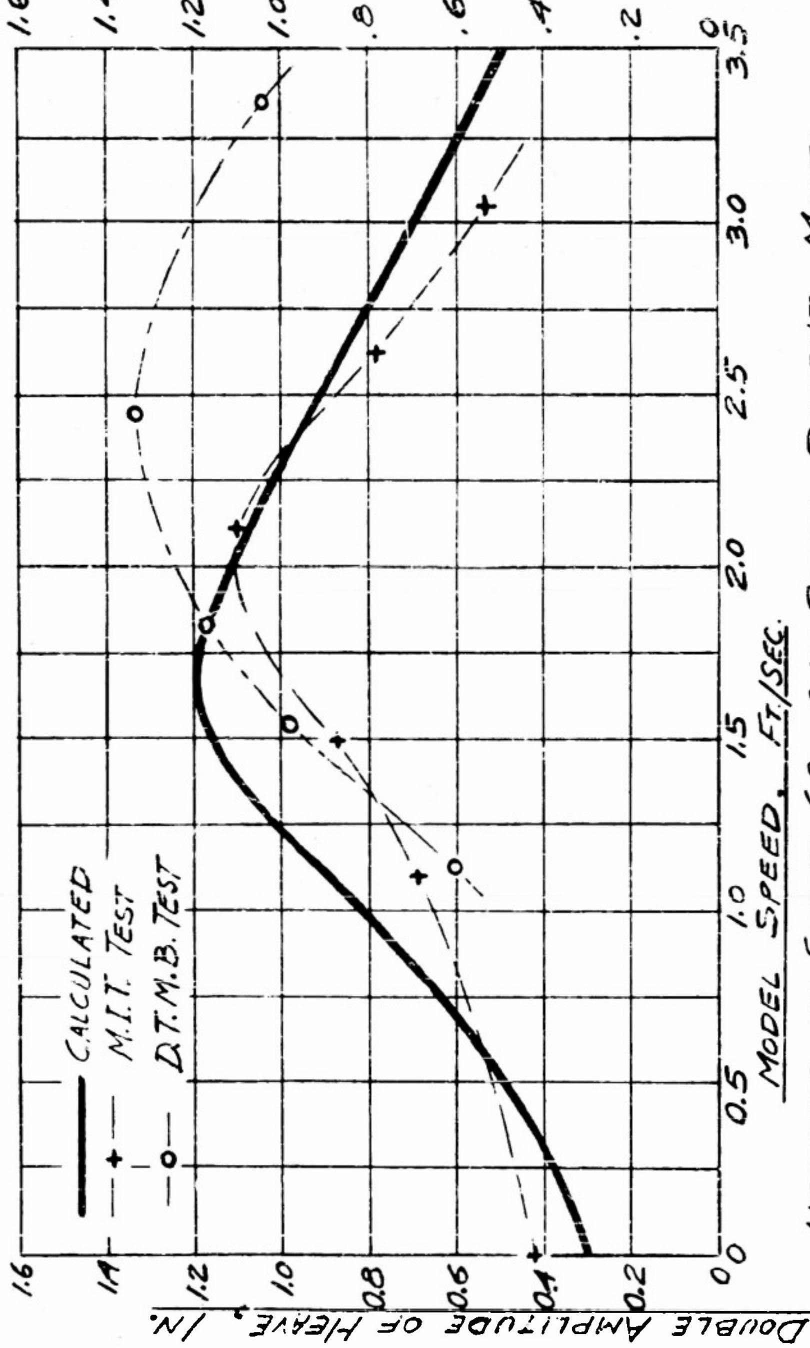


COMPUTED COEFFICIENTS OF TERMS $B\dot{z}$, $B\dot{\theta}$, $d\dot{\theta}$, AND $D\dot{z}$ VS FREQUENCY OF WAVE ENCOUNTER - SERIES 60, 0.60 BLOCK, 5-FT. MODEL

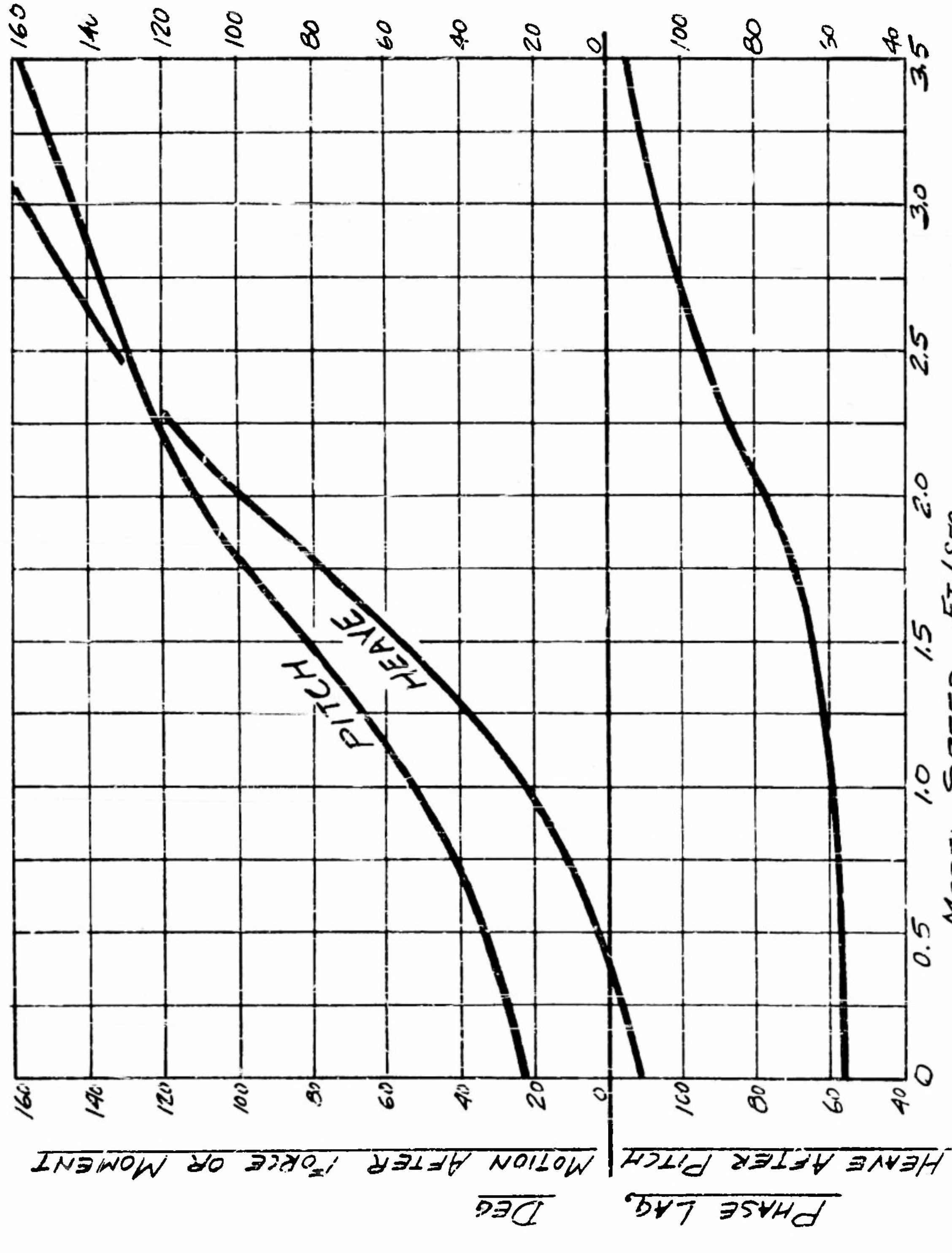


PITCHING OF SERIES 60, 0.60 BLOCK PARENT MODEL
5 FT. LONG IN 5 FT. × 1.25 IN. WAVES.

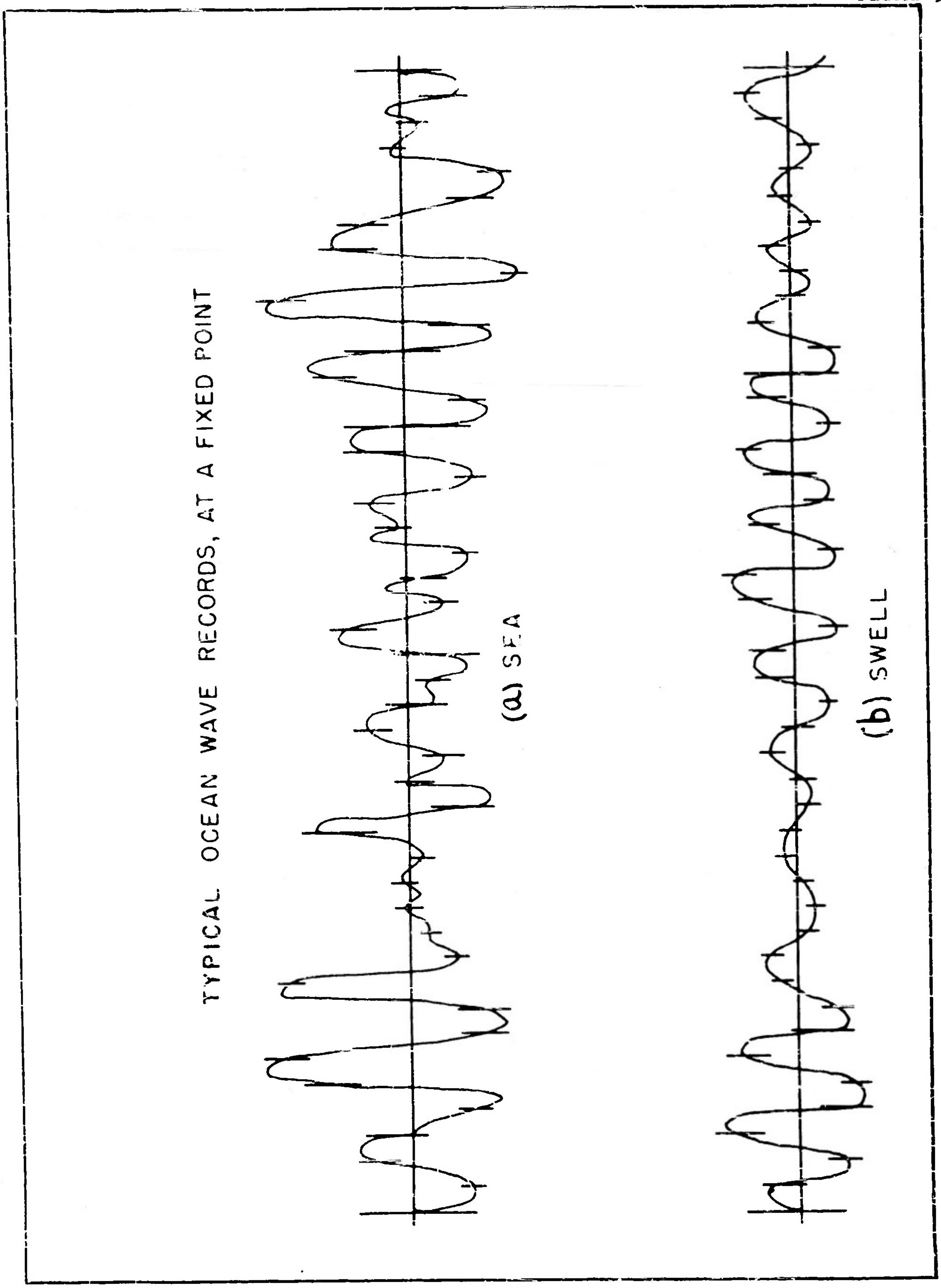


HEAVING OF SERIES 60, 0.60 BLOCK, PARENT MODEL
5 FT. LONG IN 5 FT. × 1.25 IN. WAVES

TM-106
FIGURE 4

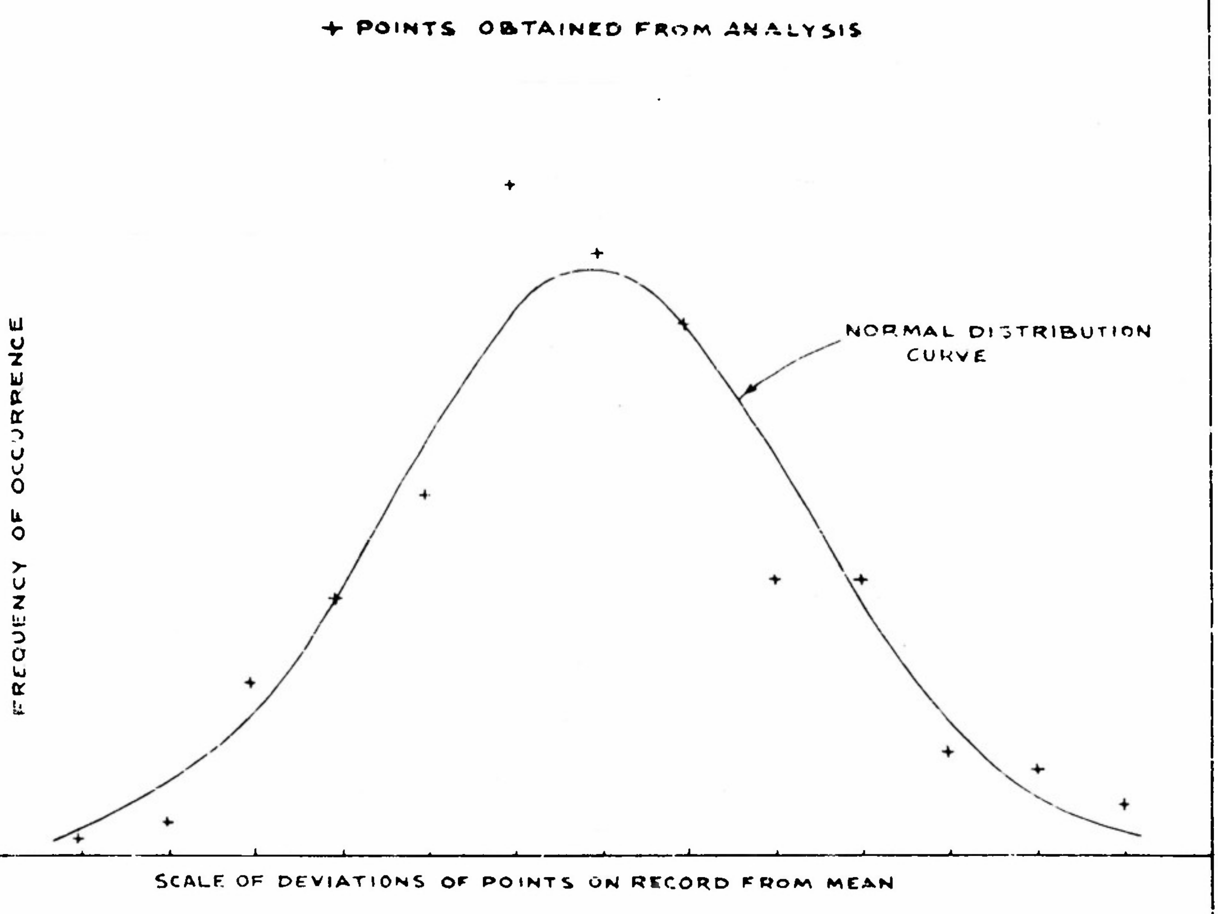


COMPUTED PHASE RELATIONSHIPS OF SERIES 60, 0.60 BLOCK, PARENT
MODEL 5FT. LONG IN 5FT. x 1.25IN. WAVES

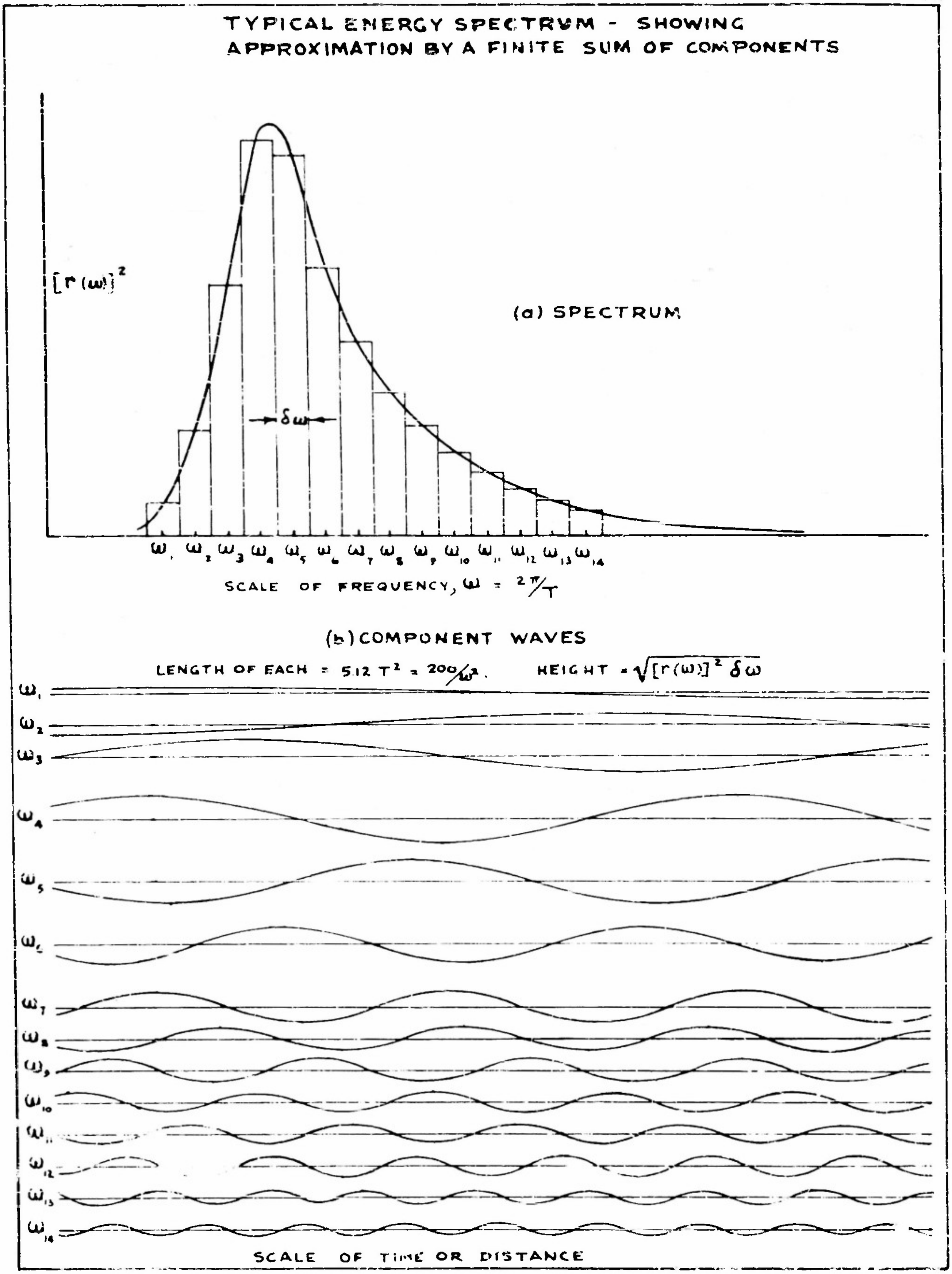

TYPICAL OCEAN WAVE RECORDS, AT A FIXED POINT
(a) SEA
(b) SWELL

TM-106
FIGURE 6

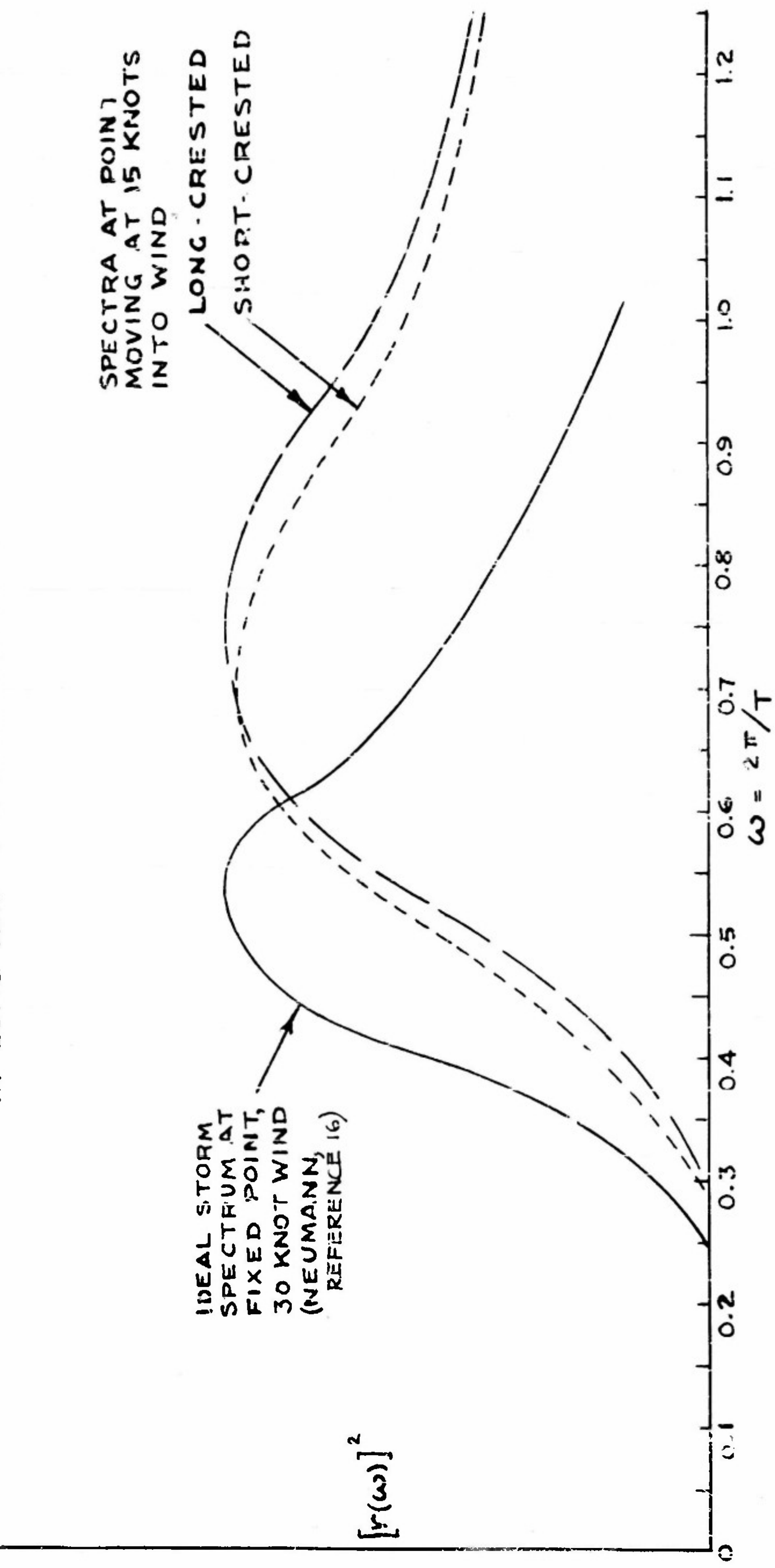
TYPICAL FREQUENCY DISTRIBUTION OF POINTS ON AN OCEAN WAVE RECORD



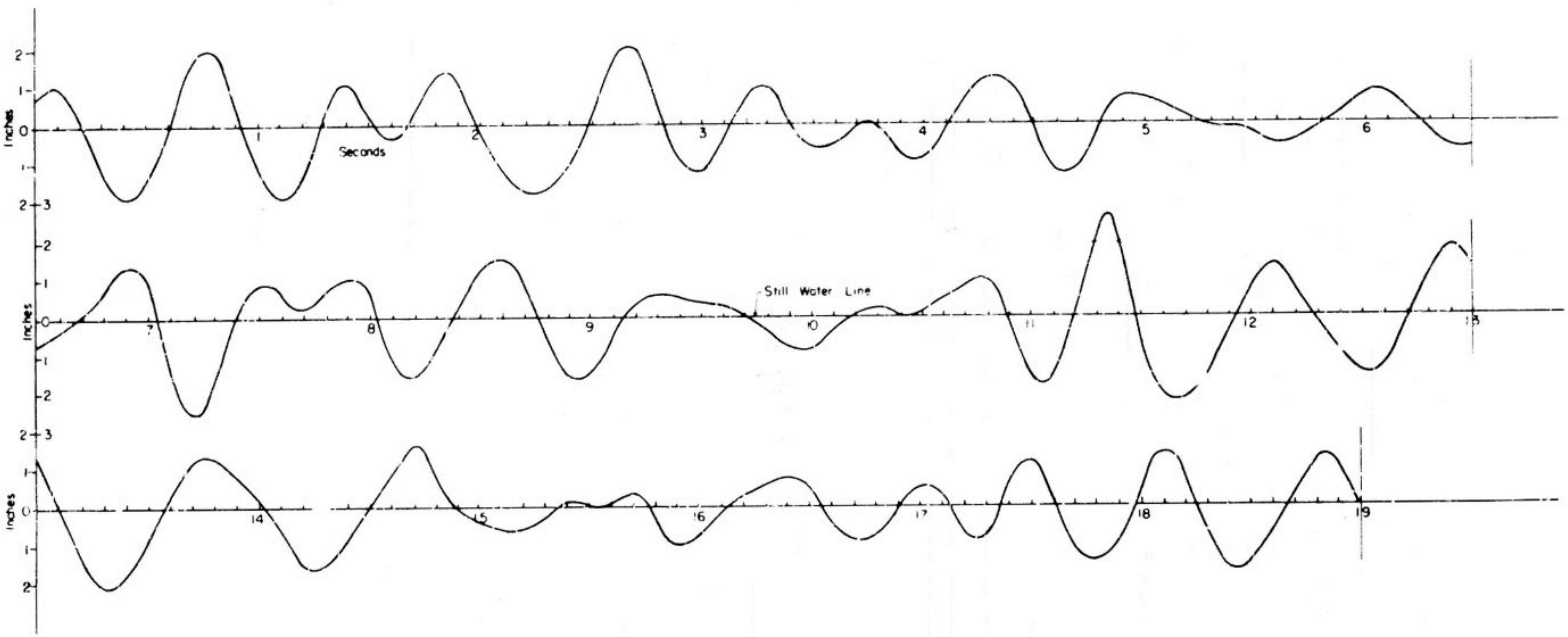
NOTE: THIS GRAPH WAS PREPARED FROM DATA GIVEN IN REF. 14, BASED ON 200 POINTS IN A 25-MIN. RECORD OBTAINED FROM A PRESSURE-TYPE WAVE RECORDER.


TYPICAL ENERGY SPECTRUM - SHOWING APPROXIMATION BY A FINITE SUM OF COMPONENTS
$[r(\omega)]^2$
(a) SPECTRUM
$\delta\omega$
ω_1 ω_2 ω_3 ω_4 ω_5 ω_6 ω_7 ω_8 ω_9 ω_{10} ω_{11} ω_{12} ω_{13} ω_{14}
SCALE OF FREQUENCY, $\omega = 2\pi/T$
(b) COMPONENT WAVES
LENGTH OF EACH = $5.12\,T^2 = 200/\omega^2$. HEIGHT = $\sqrt{[r(\omega)]^2\,\delta\omega}$
ω_1
ω_2
ω_3
ω_4
ω_5
ω_6
ω_7
ω_8
ω_9
ω_{10}
ω_{11}
ω_{12}
ω_{13}
ω_{14}
SCALE OF TIME OR DISTANCE

TM-106
FIGURE 8

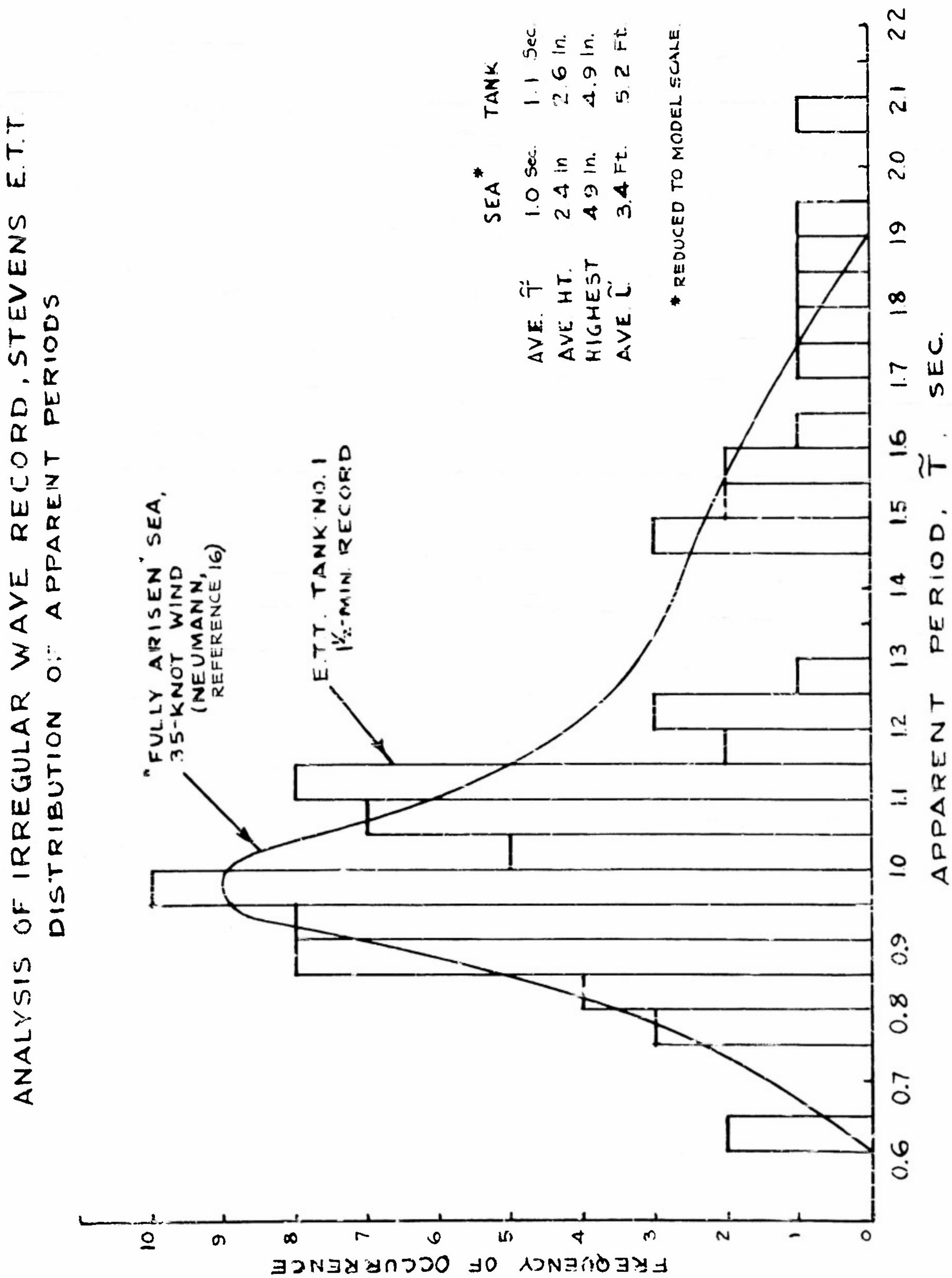


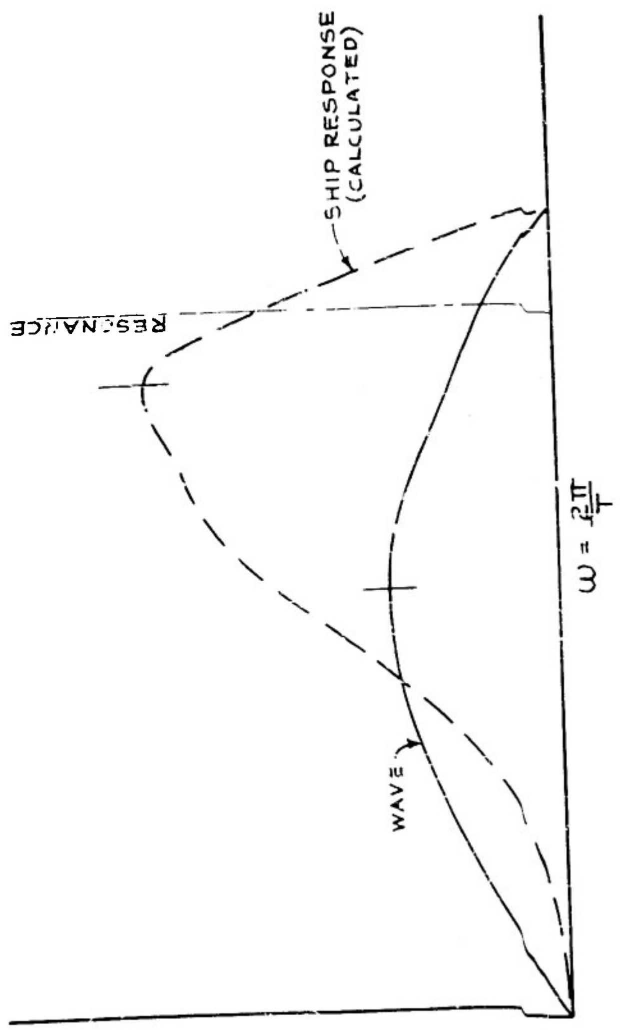
COMPARISON OF POWER SPECTRA AT FIXED AND MOVING POINTS IN LONG- AND SHORT-CRESTED SEAS



PORTION OF RECORD OF IRREGULAR TANK WAVE
RECORDED AT POINT ON CARRIAGE MOVING AT 1.42 KNOTS

TM-106
FIGURE 10





AMPLITUDE SPECTRA OF MOTION AND PITCHING

FROM FIGURE 14 OF REFERENCE 13